Two Small Improvements on Ramsey Numbers

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Abstract

We give witnesses for $r(5,22) \geq 492$ and $r(6,21) \geq 884$.

Previously, we knew $r(5,22) \geq 485$ and $r(6,21) \geq 878$, see Table IIa in revision 15 of [1]. Here, we improve this to $r(5,22) \geq 492$ and $r(6,21) \geq 884$. This is the result of a primitive computer search which the author conducted with some spare CPU time.

Let Γ be a permutation group acting primitively on $\{1,...,v\}$. The stabiliser Γ_1 of 1 partitions $\{1,\ldots,v\} = \{1\} \cup O_1 \cup \ldots \cup O_\ell$ into some $\ell+1$ orbits. We can define a vertex-transitive graph G by defining the adjacency of 1 as a subset of $\{O_1, O_2, \ldots, O_\ell\}$. For small v this process can be automated (e.g. using Magma or GAP which possess primitive group libraries). We used this to look for (K_m, K_n)-free graphs, using GAP's package grape to check for cliques. In this search, we mostly rediscovered existing lower bounds on $r(m,n)$. Here are the two exceptions.

For $r(5,22) \geq 492$, a witness comes from $\Gamma = C_{491} \rtimes C_{70}$ and the union of two orbits. Maybe the following representation is more convenient: The vertex set is $\{0,\ldots,490\}$. Two vertices x and y are adjacent if $y = 2^{7w+z} + x \pmod{491}$ for any $w \in \{0,\ldots,69\}$ and $z \in \{0,1\}$.

For $r(6,21) \geq 884$, a witness comes from $\Gamma = C_{883} \rtimes C_{98}$ and the union of two orbits. Again, a more explicit representation: The vertex set is $\{0,\ldots,883\}$. Two vertices x and y are adjacent if $y = 2^{9w+z} + x \pmod{883}$ for any $w \in \{0,\ldots,97\}$ and $z \in \{1,4,7\}$.

We want to remark that symmetry reducing techniques are essential for checking K_{22}- and K_{21}-freeness in both cases. Using grape's internal function with its symmetry reduction, this took us about $3m$ and $1h$, repectively.

References

[1] S. P. Radziszowski. Small Ramsey numbers. *Electron. J. Combin.*, 1:Dynamic Survey 1, 30, 1994.

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